Urban sprawl occurence under spatially varying agricultural bid-rent and amenities

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Abstract

This paper presents a spatially explicit model to examine the importance of agricultural amenities as a determinant of the urban and suburban spatial structure. By introducing endogenous agricultural amenities into the classical monocentric model, we provide an intuitive explanation of leapfrog development. We show how urban development patterns highly depend on the intensity of surrounding farms and their ability to produce amenities. Finally, we show how land tax policies could curb urban sprawl under certain conditions on households' preferences and farming.

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1 Introduction

Although its importance first became apparent in the second half of the 20th century, urban sprawl is still considered to be a major problem today. A consensus exists that this phenomenon is a multidimensional concept that can be examined from several different points of view. Overall, sprawl refers to the spreading outwards of a city to its outskirts. Brueckner *et al.* (2001) defined it as "*spatial growth of cities that is excessive relative to what is socially desirable*". Most observers seem to agree that fragmentation or leapfrog development is the most significant feature of urban sprawl. EEA (2006) stresses that "*urban sprawl is the physical pattern of low-density expansion of large urban areas, under market conditions, mainly into the surrounding agricultural area, (...), development is patchy, scattered and strung out, with a tendency for discontinuity. It leapfrogs over areas, leaving agricultural enclaves*".

The purpose of this paper is to investigate the conditions required for leapfrog development. We present a spatially explicit model which highlights the importance of agricultural amenities to determine suburban spatial structure. By introducing endogenous agricultural amenities into the classical monocentric model, we offer an intuitive explanation of the interplay between agriculture and urban sprawl.

At the urban fringe, cities and agriculture compete for land. In recent years, most urban development has occurred on agricultural land. While planning and zoning policies play an important role in controlling the conversion of agricultural land, the general trend is for the large majority of

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urbanised land to have been converted from agricultural uses. Often, it is the best quality agricultural land that is the first to be converted to urban use (Greene & Stager (2001), Walker (2001) and Burchfield *et al.* (2006)). As well as often possessing less suitable physical qualities for development, natural areas and woodlands are more resistant to urbanisation because they can be protected by strict legislation. For a city to grow spatially, households must be able to bid away additional land from farmers. A successful bid by a household means that the land is worth more for urban, rather than for agricultural, use. Thus, land conversion is basically guided by economic mechanisms, directing resources to their best uses.

The economic literature has widely studied the trade-offs between agriculture and urbanisation. Initially, several studies used bid-rent models originating from von Thünen (Walker & Solecki (2004)). The confrontation of the bid-rent functions of both farmers and urban households, determines the size and the boundary of the city (Wheaton (1974)). The fundamental forces underlying the expansion of cities are clearly stated by the monocentric-city model[1], which focuses on the trade-off between space consumption and the accessibility of the city center.

To examine urban sprawl within the monocentric-city model, several articles have considered the anticipation of future spatial growth (Mills (1981), Wheaton (1982), Titman (1985) and Capozza & Helsley (1989)). These studies argue that amenities are spatially homogeneous. The existence of scattered urban areas is explained by the expectation behaviour of owners. However, when amenities are considered as spatially heterogeneous, it is possible to observe a scattered urban development. This is due to the fact that the household bid-function is not necessarily monotonic with regard to the distance from the central business district (CBD) (Polinsky & Shavell (1976), Ogawa & Fujita (1980), Yang & Fujita (1983), Fujita & Kashiwadani (1989)). In their model, Polinsky & Shavell (1976) include an environmental amenity characterised by its distance to the CBD and show how the amenity changes the spatial pattern of property values. In these studies, the total developed area is not fragmented and the agricultural rent is always exogenous.

According to the monocentric city model, a household chooses the residential location that provides the best trade-off between land costs and commuting costs. Thus, we expect to find wealthier households living far from the CBD. To explain the fact that, in some cities poorer people live near the city center, while the rich live on the periphery, Brueckner *et al.* (1999) introduce some additional factors to the monocentric city model including the availability of new suburban housing, transportation modes, suburban zoning and amenities. Brueckner *et al.* (1999) include amenities characterised by distance to the CBD to determine the location of different income classes. In this paper, the environmental amenity does not occupy space. In contrast, Mills (1981), Nelson (1985), and Lee & Fujita (1997) analyse the effects of "greenbelts" that form a ring of open space around a city and which are characterised by their distance from the CBD.

Several papers develop two-dimensional urban models including environmental amenities that show the effect of the location, size and shape of open space on equilibrium housing, land prices, and city boundaries in an open-city model (Wu & Plantinga (2003); Wu (2006)). Wu & Plantinga (2003) show that the designation of open space around a city can lead to leapfrog development. Wu (2006) demonstrates how development patterns and community characteristics are influenced by the spatial distribution of environmental amenities. These studies provide a more intuitive explanation for leapfrog development than previous studies, but still treat agricultural rent and amenities as exogenous.

Generally, theoretical models consider only agricultural or residential land use, with the exception of Muth (1961), who describes the movement of city limits in relation to an agricultural

[1] The model was developed by Alonso (1964), Muth (1969) and Mills (1972). See Fujita (1989) for a technical overview.

hinterland lying beyond the city. Muth (1961) analyzes relationships between the city boundary and several economic variables, including wage rates and the relative demand characteristics of housing and agricultural products. More recently, Walker (2001) and Cavailhès *et al.* (2004) present a model treating agricultural and urban land uses simultaneously. Both studies borrow ideas from the monocentric-city model and the agricultural model developed by von Thünen. Walker (2001) discusses several aspects of land cover change dynamics resulting from economic development and the interplay of urban and agricultural processes. Introducing rural amenities produced by farmers, Cavailhès *et al.* (2004) demonstrate the existence conditions of a suburban area, where farmers and households share space. These studies were not specifically concerned with urban sprawl, but offer an interesting analytical framework for better understanding the interactions between the city and agriculture.

This short review of the literature allows us to identify three main lessons. First, monocentric city models, exploring the possibilities of leapfrog development, assume an exogenous agricultural rent to define the city boundary. By doing so, these studies are not able to explain the interactions between agriculture and cities. Thus, farm structures have no effect on agricultural land conversion. Second, the literature shows the importance of amenities in explaining urban sprawl. Amenities may be exogenous or endogenous. Exogenous amenities are provided by natural features in the landscape. Endogenous amenities are provided by human activities, such as local public services and agriculture. Third, apart from Cavailhès *et al.* (2004), no study explicitly models agricultural amenity. However, Cavailhès *et al.* (2004) considered that the amenity is proportional to the agricultural area, which does not reflect the nature of the farm. According to their model, extensive farms produce the same level of amenities as intensive agriculture.

This paper seeks to fill these gaps by proposing a theoretical model that combines a monocentric city model with a von Thünen model. Our model is closely related to Wu (2006) and Cavailhès *et al.* (2004). Contrary to Wu (2006), we model the behavior of farmers *à la* von Thünen. Small and intensive farms are located close to the city boundary while larger, more extensive farms are further away. This can be explained by the urban pressure on agricultural land prices. Far away from the city boundary, land becomes less expensive and may be substituted to capital. This may occur within a few miles for small cities, and up to ten miles away for larger settlements (Cavailhès & Wavresky (2007)). Our model emphasises the role of agricultural amenities in household welfare. We assume that agricultural amenities are a joint-product of farm production. Unlike Cavailhès *et al.* (2004), we assume that the level of amenity is defined at each point in space according to the level of agricultural intensity. So, intensive farms produce fewer amenities than extensive ones.

The remainder of the paper is organised as follows. In section 2, we present the model and discuss the conditions for spatial equilibrium and more particularly for leapfrog development. Section 3 gives a numerical illustration of the main results of the model, while in section 4 we expand our model to include the case of the introduction of a land tax policy. Section 5 concludes the paper.

2 The model

2.1 Structure of the city

Space is represented by the real line $X = (-\infty, +\infty)$ with a CBD at its origin. It is assumed that all non-agricultural employment is concentrated in the CBD. There are three types of agents competing in the land market: N_u identical urban households working in the CBD, N_p identical periurban households working in the CBD and N_a identical farmers. We assume that all land is owned by absentee landlords. Our city is open, meaning that migration in and out is possible.

The farmers' bid function To produce Y, farmers use two kinds of input: land (L) and non-land inputs (K)[2]. The production function is given by $Y = F(K, L)$. This function is increasing and concave in each of its arguments and has constant returns to scale, which implies $y = Y/L = f(k)$, where y is the output per hectare and k is non-land input per hectare[3]. We assume $f(0) = 0$, $f'(k) > 0$ and $f''(k) < 0$.

Each farm sells its products at the local market within the CBD. We assume that transportation costs are proportional to the distance to the CBD x and that t is the cost per unit of distance. Crops, land and non-land inputs are available without restriction at competitive prices, respectively p, r_a and p_k.

The farmers' profit function is:

$$\pi(k, x) = (p - tx) f(k) - p_k k - r_a \tag{1}$$

Profit maximisation with respect to k implies that $(p - tx) f'(k) = p_k$

If $f(k)$ is a Cobb-Douglas then $f(k) = Ak^\alpha$ with $0 < \alpha < 1$, we obtain:

$$k^* = \left[\alpha \frac{A(p - tx)}{p_k}\right]^{\frac{1}{1-\alpha}} \tag{2}$$

and

$$y^* = A\left[\alpha \frac{A(p - tx)}{p_k}\right]^{\frac{\alpha}{1-\alpha}} \tag{3}$$

Eq. (2) shows that k falls with increasing distance away from the city. Far from the city, farmers substitute non-land inputs in favour of land inputs. We note that k reaches zero at $\underline{x} = p/t$. This entails that output (Eq (3)) is a decreasing function of the distance to CBD and equals zero at a critical distance $\underline{x}$.

Since $f(.)$ has constant returns to scale, in equilibrium, all farmers make zero profit per unit of area at each x, that is:

$$r_a^*(x) = A(1 - \alpha)\left(\frac{\alpha A}{p_k}\right)^{\frac{\alpha}{1-\alpha}} (p - tx)^{\frac{1}{1-\alpha}} \tag{4}$$

We note that $r_a(x)$ is a decreasing function of distance and equals zero beyond $\underline{x}$. Agricultural activities are influenced by the distance that separates them from the city. Near to cities, farms tend to be intensive. The more intensive farms are, the higher the ratio k. Far away from cities, farms become progressively more extensive as the ratio k falls.

Urban households Within the city, there is no agriculture. We assume that urban households are insensitive to agricultural amenities (described later on). Formally, each houshold chooses a combination of residential space q_h, location x, and a numeraire of non-housing goods s to maximise their utility $U(s, q_h)$ subject to the budget constraint:

$$\max_{q_h, s, x} U[s, q_h] \qquad \text{s.t. } w = r_u(x) q_h + s + \tau x \tag{5}$$

where w is the gross household income and τ is the round-trip commuting cost per kilometre. $r_u(x)$ is the housing price at x. We use a Cobb-Douglas specification of the utility function :

[2] The non-land inputs represents the agricultural inputs such as gears, seeds, fertilizers and all other equipments used in different agricultural activities.

[3] This specification is inspired by Beckmann (1972)

$U\left[s,q_h\right]=q_h^{\beta}s^{1-\beta}$ (with $0<\beta<1$). The first-order conditions for the utility maximisation problem define the optimal choice of housing space and non-housing goods at each location:

$$s^{*}=\left(1-\beta\right)\left(w-\tau x\right) \tag{6}$$

$$q_h^{*}=\frac{\beta\left(w-\tau x\right)}{r_u\left(x\right)} \tag{7}$$

Eqs. (6) and (7) show that expenditures on housing and composite goods fall with increasing distance x. For each unit of distance from the CBD, total expenditures on total housing fall by βt and expenditures on the composite good fall by $\left(1-\beta\right)t$.

The indirect utility function V_u of the household living in the city is given by:

$$V_u=\frac{\beta^{\beta}\left(1-\beta\right)^{1-\beta}\left(w-\tau x\right)}{r_u\left(x\right)^{\beta}} \tag{8}$$

The bid-rent for each household at any location is given by:

$$r_u^{*}\left(x\right)=\left[\frac{\beta^{\beta}\left(1-\beta\right)^{1-\beta}\left(w-\tau x\right)}{\overline{V_u}}\right]^{\frac{1}{\beta}} \tag{9}$$

The bid-rent function (9) corresponds to the household's maximum willingness to pay for housing at location x. At equilibrium, households are indifferent to where they locate because their utility $\overline{V_u}$ is the same at each location within the city..

We define $\rho_u(x)$ as the density of urban households at each location (i.e. number of households per hectare). We have:

$$\rho_u(x)=\frac{D(x)}{q_h^{*}(x)} \tag{10}$$

where $D(x)$ is the land distribution (residential lot size per hectare of land) and $q_h^{*}(x)$ is the optimal lot size (residential lot size per household). Without loss of generality, we set $D(x)=1$[4].

Periurban households By selecting a residential location outside the city, periurban households are also choosing a rental price and a level of agricultural amenities. The detailed description of agricultural amenities is treated in the next section, we first focus on the periurban households' trade-off. Formally, each household chooses a combination of residential space q_h, location x, level of agricultural amenities $a(x)$, and a numeraire of non-housing goods s to maximise their utility $U\left(s,q_h,a\left(x\right)\right)$ subject to the budget constraint. We use a Cobb-Douglas specification of the utility function: $U\left[s,q_h,a\left(x\right)\right]=q_h^{\beta}s^{1-\beta}a\left(x\right)^{\gamma}$ (with $0<\gamma<1$). We obtain the same first-order conditions as for urban households (Eqs. (6) and (7)). However, the indirect utility function V_p of the household living outside the city is given by:

$$V_p=\frac{\beta^{\beta}\left(1-\beta\right)^{1-\beta}\left(w-\tau x\right)}{r_p\left(x\right)^{\beta}}a\left(x\right)^{\gamma} \tag{11}$$

[4] $D(x)>1$ conveys the existence of configurations where there are several residential lots at one given location (the case of buildings for example). As we don't focus on the housing density distribution within the city, we can, without loss of generality, set $D(x)$ to one for simplification.

The bid-rent for each household at any location is given by:

$$r_p^*(x) = \left[\frac{\beta^\beta (1-\beta)^{1-\beta} (w - \tau x)}{\overline{V_p}}\right]^{\frac{1}{\beta}} a(x)^{\frac{\gamma}{\beta}} \tag{12}$$

The bid-rent function (12) reveals the difference between urban and periurban households. In the case of the urban area (9), there are no agricultural amenities but households enjoy a level of urban amenities normalised to one ($a_u(x) = 1$) and equally distributed across the whole city. Urban bid-rent always falls with the distance away from the CBD to compensate residents for the costs of commuting. However, with spatial variation in agricultural amenities, the pattern of housing prices is more complicated. Clearly, periurban households decide to live in rural areas if $a(x) > 1$ and they must be compensated for the loss of urban amenities and higher level of commuting costs. In this case, agriculture must therefore produce a sufficient level of amenities to attract households.

Similarly to Eq.(10), we set $\rho_p(x)$ as the periurban households density at location x.

At equilibrium, the level of the indirect utility function will be the same for both urban and periurban households ($\overline{V_u} = \overline{V_p} = \overline{V}$).

Agricultural amenities To improve the land resource, farms carry out stewardship practices such as the maintenance of hedges and tracks, drainage, erosion control, and crop rotation. These practices also have the advantage of providing a range of environmental goods and services. These positive externalities of production can be considered as agricultural amenities, which may be highly valued by periurban residents (Huylenbroeck (1999)).

Insofar as agriculture has an undeniable spatial dimension, we can deduce that the spatial distribution of agricultural amenities is not an exogenous phenomenon. Cavailhès & Wavresky (2007) show that, in the French context, small, intensive farms are located close to cities and larger, extensive farms are located further away. They explain this as a result of the pressure that the adjacent urban area exerts on agricultural land prices, making farmers substitute land for non-land inputs (e.g. compound feeds, chemical fertilisers, pesticides, and increased mechanisation). Without loss of generality, we consider agricultural amenities as a net balance of positive externalities (e.g. landscape quality, biodiversity) and negative ones (e.g. pollution, nuisances). More extensive farms provide a higher level of agricultural amenities in the sense that their crop management favour the joint-production of positive externalities and that fewer non-land inputs lowers negative externalities. In our spatialised economy, farms located near the city provide a low level of amenities because they use more non-land inputs per hectare. As we get further away from the urban area, farming gets more extensive and the joint-production of agricultural amenities increases (see Fig. 1). This assumption of an increasing flow of agricultural amenities far from the CBD is consistent with observations made in urban ecology showing that there is a positive gradient of species-richness and ecosystem complexity at distance from the urban area (Hansen *et al.* (2005), McKinney (2006), Czamanski *et al.* (2008)).

As a by-product of agricultural activities, the amenity level depends on the ratio k. Note that farmers don't take amenities into account in their behaviour as they are externalities and they are not paid for their production. However, these amenities are valued by periurban households who make their trade-off between accessibility to the CBD and residential space consumption. The level of agricultural amenities impacts this trade-off in the sense that periurban households may wish to bid more in areas where amenities are high enough. However, by converting agricultural land to residential use, periurban households also destroy agricultural amenities. Urban sprawl therefore reaches its limits in the destruction of valued landscapes and agricultural environment, justifying

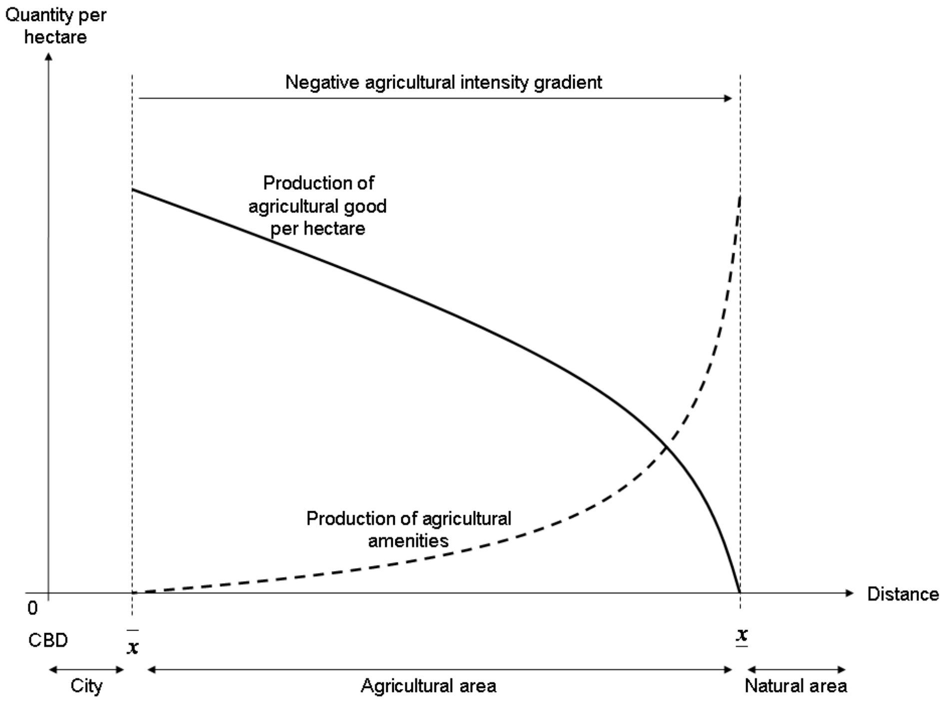

Figure 1: Trade-off between agricultural production and amenities

the introduction of a negative externality between periurban households (Irwin & Bockstael (2002)). This is introduced in the model making use of the fact that at any location, available land is unity. Noting that the periurban space is shared between periurban households and farmers, we have:

$$\rho_p(x)q_h(x) + \rho_a(x)L(x) = 1 \tag{13}$$

where $\rho_p(x)$ and $\rho_a(x)$ are respectively the density of periurban households and of farmers at x, and $q_h(x)$ and $L(x)$, their land consumption. We define $\Theta(x)$ as the fraction of land dedicated to agricultural use:

$$\Theta(x) = \rho_a(x)L(x) = 1 - \rho_p(x)q_h(x) \tag{14}$$

We consider the following specification of the amenity distribution outside of the city:

$$a(x) = \delta\frac{\Theta(x)}{k(x)} \tag{15}$$

where δ is a positive constant which can be interpretated as the capacity of a given type of agriculture to provide amenities, or in other words, the degree of jointness between amenity production and agricultural activity. For example, we can imagine that some crops, such as single-crop farming or intensive soilless livestock farming, could be considered to have a low capacity to provide amenities valued by households (Palmquist *et al.* (1997)). On the other hand, viticulture, extensive livestock farming or fruit horticulture can produce more valued amenities and would therefore have a higher δ (Le Goffe (2000), Irwin (2002)).

As we move away from the CBD, the amenity level gets higher as farms become more extensive (i.e. with lower k). Without any regulation, households could outbid farmers and agriculture amenities would be destroyed when new households would come and locate, as in Hardin's tragedy of the commons. Introducing $\Theta(x)$ in $a(x)$ allows us to take into account a negative externality between periurban households. When $\Theta(x)$ increases, meaning a larger share of agriculture, the level of amenities increases. On the contrary, if the share of residential use gets more important, $\Theta(x)$ decreases and the level of amenities gets lower.

2.2 Spatial equilibrium conditions

After deriving the behavioural functions for farmers and households, we present the conditions for the existence of the city and leapfrog development.

Existence conditions of the city The city is the area where urban households live. Let $\overline{x}$ be the boundary of the city. Land being rented to the highest bidder, the city can be represented by the set C:

$$C = \{x < \overline{x} \mid r_u^*(x) > r_a^*(x)\} \tag{16}$$

The location of the urban fringe $\overline{x}$ is given by:

$$r_u^*(\overline{x}) = r_a^*(\overline{x}) \tag{17}$$

$\overline{x}$ exists if the households and farmers' bid-rent functions intersect at least once within the interval $[0, \underline{x}]$. To reach this possibility, parameters of the models must obey the following conditions:

$$w > \Omega p^{\frac{\beta}{(1-\alpha)}} \text{ and } \frac{w}{\tau} < \frac{p}{t} \tag{18}$$

where Ω is a positive constant (see Appendix A for details). These conditions mean that households income must be large enough, relative to the prices of agricultural products, and that the trade-off between urban and agricultural land use can only be made within the interval $[0, \underline{x}]$, as from $\underline{x} = \frac{p}{t}$, all agricultural activity stops and land is occupied by natural areas.

The city boundary is thus defined by the competition between urban households and farmers for land. However, the presence of another agent, the periurban household, allows us to consider the possibility of leapfrog development, previously defined as a scattered pattern of urban sprawl.

Leapfrog conditions To determine the conditions for the occurence of leapfrog development, we investigate the properties of a periurban area, where the space is shared between farmers and households, meaning that they have the same bid-rent. Let P be such an area. It is defined by:

$$P = \left\{x \in [\overline{x}, \underline{x}] \mid r_p^*(x) = r_a^*(x)\right\} \tag{19}$$

Leapfrog development occurs if and only if P is not simply connected to C. Intuitively, leapfrogs occurs in areas disjoint from the existing developed area. Clearly, leapfrog development occurs if there exists $x_1 < x_2 < \underline{x}$, so that:

- for all $x \in [\overline{x}, x_1] \cup [x_2, \underline{x}]$, we have $r_a^*(x) > r_p^*(x)$, meaning agricultural use only.
- for all $x \in [x_1, x_2]$, we have $r_p^*(x) = r_a^*(x)$, meaning a mixed land use with both farmers and periurban households.

We have the following condition for an intermediate area dedicated to agricultural use only (see Appendix B for details).

$$a(\overline{x}) > 1 \tag{20}$$

(x_1, x_2) are endogenously determined by the model, using $r_a^*(x)$ and $r_p^*(x)$ as specified in Eqs. (4) and (12) respectively and the amenity level valued by periurban households being given by $a(x) = \frac{\delta}{k(x)}$ (from Eq. (15) with $\Theta(x) = 1$, as all space is originally occupied by farmers only).

Characteristics of the leapfrog area Once x_1 and x_2 are determined, we define the density characteristics within the periurban area. Note that the density of periurban households will not change x_1 and x_2. As soon as a single periurban household moves in the periurban area, we have $\Theta(x) < 1$ and the amenity level is lowered (Eq. (15)). If the periurban area was completely built, we would then have $\rho_p(x)q_h(x) = 1$ and $\Theta(x) = 0$, meaning no amenities and $r_p^*(x) = 0$ (Eq.(12)). This illustrates that if the level of amenities is too low, notably because too many periurban households move in, their reservation utility would not be reached and they would migrate to other cities. The complete urbanisation of the periurban area is therefore not possible in our model. Consequently, a mixed land use in our leapfrog area implies $\Theta(x) \in]0,1[$.

Recalling that $r_p^*(x) = r_u^*(x)a(x)^{\frac{\gamma}{\beta}}$ and using (19), we can define the optimal amenity distribution inside the leapfrog area:

$$a^*(x) = \left(\frac{r_a^*(x)}{r_u^*(x)}\right)^{\frac{\beta}{\gamma}} \tag{21}$$

We derive the optimal periurban households density within the leapfrog area, from Eqs. (14), (15) and (21):

$$\rho_p^*(x) = \frac{1}{q_h^*(x)}\left[1 - \frac{k^*(x)}{\delta}\left(\frac{r_a^*(x)}{r_u^*(x)}\right)^{\frac{\beta}{\gamma}}\right] \tag{22}$$

Definition of spatial equilibrium We sum up spatial equilibrium for land developement. Land is rented to the highest bidder, thus spatial equilibrium is reached if the following conditions are satisfied:

1. The spatial equilibrium is given by the prevailing land rent at x:

$$r^*(x) = \max\left\{r_u^*(x), r_p^*(x), r_a^*(x)\right\} \tag{23}$$

where $r_u^*(x)$ is given by Eq.(9), $r_p^*(x)$ is given by Eq.(12) and $r_a^*(x)$ is given by Eq.(4).

2. At equilibrium, the delimitations of the different areas is characterised by the city boundary $\overline{x}$ given by (17), the limits of the leapfrog area (x_1, x_2) determined by the relative position of $r_a^*(x)$ and $r_p^*(x)$, and the limits of the agricultural area $\underline{x} = p/t$.

3. The urban area (resp. periurban area) must be sufficient to provide housing for all urban (resp. periurban) households who have chosen to settle in (resp. outside of) the city. The number of urban households, periurban households and farmers is endogenously determined by the model and given by:

$$\int_0^{\overline{x}} \rho_u^*(x)dx = N_u \quad ; \quad \int_{x_1}^{x_2} \rho_p^*(x)dx = N_p \quad ; \quad \int_{\overline{x}}^{\underline{x}} \rho_a^*(x)dx = N_a \tag{24}$$

where $\rho_u^*(x)$ is given using Eqs. (7) and (10) and $\rho_p^*(x)$ is given by Eq.(22). The fraction of agricultural land within the periurban area is given by[5]:

$$\Theta^*(x) = 1 - \rho_p^*(x) q_h^*(x) \tag{25}$$

Detailed calculations for $\rho_p^*(x)$ and $\Theta^*(x)$ within the periurban area are available in Appendix C.

4. At equilibrium, the level of agricultural amenities $a^*(x)$ is given by:

$$a^*(x) = \begin{cases} 0 & \text{if } x \in [0, \overline{x}] \\ \left(\frac{r_a^*(x)}{r_u^*(x)}\right)^{\frac{\beta}{\gamma}} & \text{if } x \in [x_1, x_2] \\ \frac{\delta}{k^*(x)} & \text{otherwise} \end{cases} \tag{26}$$

3 Spatial pattern of land development

3.1 Calibration of the model

To visualise the relative positions of the bid-rent functions, we run several simulations of our city model, using observed data. Most of our data come from the Eurostat database[6] and from the Farm Accountancy Data Network[7]. According to Eurostat, between 2000 and 2010, the average income for French households was 19 342 €. Around a quarter of their total consumption is dedicated to housing (26,3% in 2003), and around 6% is allocated to the use of private cars and public transport. As for the agricultural parameters, the FADN gives an average level of charges of 1 747 €/ha per farm between 2002 and 2009, and an average gross product of 2 141 €/ha per farm, for the same period. The share of non-land costs per hectare is estimated at approximatively 90% of total costs.

We set our parameters as specified in Table 1. However, not all of our parameters can be found in databases (ie. δ, γ, V and t). Therefore, we must calibrate the model, by testing it under different specifications of given parameters.

For calibration purposes, we look at how the model behaves along with the parameters.

The first parameter we analyse is δ, which can be interpretated as a technical parameter representing the capacity of a farm to provide amenities. Simulations were first made by changing δ at a given level of $\{\gamma, V, t\}$. From (12) and (15), we have $\partial r_p/\partial\delta > 0$, an increase of δ should increase the periurban bid-rent. The urban and agricultural bid-rent both remain unchanged. Fig.2a shows how the size of the leapfrog area ($x_2 - x_1$) evolves as δ changes. Three possibilities for spatial development patterns arise. The first case consists in the absence of any sprawl development. In this case, $\delta < \delta_{\min}$, meaning that farms provide a level of amenities too low to convince periurban households to locate. The second possibility is the emergence of leapfrog development ($\delta_{\min} < \delta < \delta_{\max}$). Farms provide sufficient agricultural amenities to persuade periurban households to relocate. Leapfrog development then occurs, i.e. the periurban area becomes disconnected from the city. Between $\delta_{\min}$ and $\delta_{\max}$, the leapfrog urban area gets larger as δ increases. Finally, the third case is an extension of the existing urban area. In this particular situation, farms have such a high capacity to provide amenities that the periurban area eventually links up with the city ($\delta > \delta_{\max}$). The fact that, over

[5] Constant returns to scale do not allow us to determine the land demand function for farmers. We therefore can't calculate $\rho_a^*(x)$ and $L^*(x)$ separately. However, we have the fraction of agricultural land $\Theta^*(x) = \rho_a^*(x)L^*(x)$.

[6] http://epp.eurostat.ec.europa.eu/portal/page/portal/statistics/search_database (last visit: January 18th 2012)

[7] The FADN is a european database providing accountancy data on the incomes and business operation of agricultural holdings in the European Union. http://ec.europa.eu/agriculture/rica/database/database_en.cfm (last visit: January 18th 2012)

Symbol	Interpretation	Value	Source / Comment
w	household income	20 000 €	Eurostat
τ	household transport costs	1 000 €	Eurostat
β	budget share dedicated to housing	0.25	Eurostat
V	equilibrium utility level	2 500	Calibrated
p	agricutural goods price	2 000 €$/ha$	FADN
p_k	non-land inputs price	1 700 €$/ha$	FADN
t	farmers transport costs	40 €	Calibrated
A	technical constant	1	Set to one for simplification (as in Cavailhès *et al.* (2004))
α	elasticity of production factor k	0.85	FADN
δ	capacity for a farm to provide amenities	0.30	Calibrated
γ	periurban households' preferences for amenities	0.80	Calibrated

Table 1: Parameters value and signification

$\delta_{\max}$, the leapfrog development pattern disappears is due to one of the leapfrog equilibrium conditions, i.e. the amenity level at the city boundary must be less than one. If this condition is not met, then the development pattern becomes urban extension with the following succession of areas: city, periurban area and agriculture. The value of $\delta_{\max}$ can be calculated using the formula given in Appendix B. In our case, $\delta_{\max} = 0.56$ and $\delta_{\min} = 0.23$. Once the periurban area has joined the city ($\overline{x} = x_1$), its size increases more slowly until $x_2 = w/\tau$. The maximum size of the periurban area is therefore $(x_2 - x_1)_{\max} = (w/\tau) - \overline{x}$, to which $(x_2 - x_1)$ tends when δ increases.

The second parameter we consider is the preference for amenities γ. This parameter makes the difference between the urban bid-rent and the periurban bid-rent. As it represents the preferences that households have for amenities, we foresee that when it increases, households will tend to settle outside the city (from (12), $\partial r_p/\partial\gamma > 0$). Fig.2b shows the impact of γ on the size of the leapfrog area, at a given level of $\{\delta, V, t\}$. Once again, only the periurban bid-rent is changed. When preferences are lower than a threshold $\gamma_{\min}$, no sprawl occurs because periurban households have no incentive to move. In our case, we have $\gamma_{\min} = 0.57$. Over $\gamma_{\min}$, the households' preference are high enough for the associated amenities to be a sufficient trigger to move to a periurban area. The disconnected periurban area therefore gets larger as γ increases.

Making γ and δ vary simultaneously, we obtain the diagram in Fig.3. For a given (γ, δ) we can observe what the expected spatial pattern will be. As soon as δ gets higher than $\delta_{\max}$, the leapfrog development pattern will disappear, to be replaced by a fusion between the city and the periurban area, at any γ. Fig.3 allows us to refine our interpretation of δ. The parameter δ scales the ability of several agricultural types to generate positive (or negative) externalities. To illustrate this, take the example of grasslands or forests, where higher levels of production intensity do not prevent relatively high levels of amenity being produced. On the other hand, agriculture activities such as livestock farming, generate manure production and can require the construction of additional farm buildings all of which may be negatively valued by households. The level of intensity has a greater impact on the level of amenities provided. Over $\delta_{\max}$, sprawl takes the form of urban extension only because amenities at the city fringe are sufficiently high for periurban households ($a(\overline{x}) > 1$). Leapfrog occurs under two conditions: high households' preferences for amenities and an intermediate capacity of farms to generate them.

According to our model, cities surrounded by highly amenity generating farming (for example,

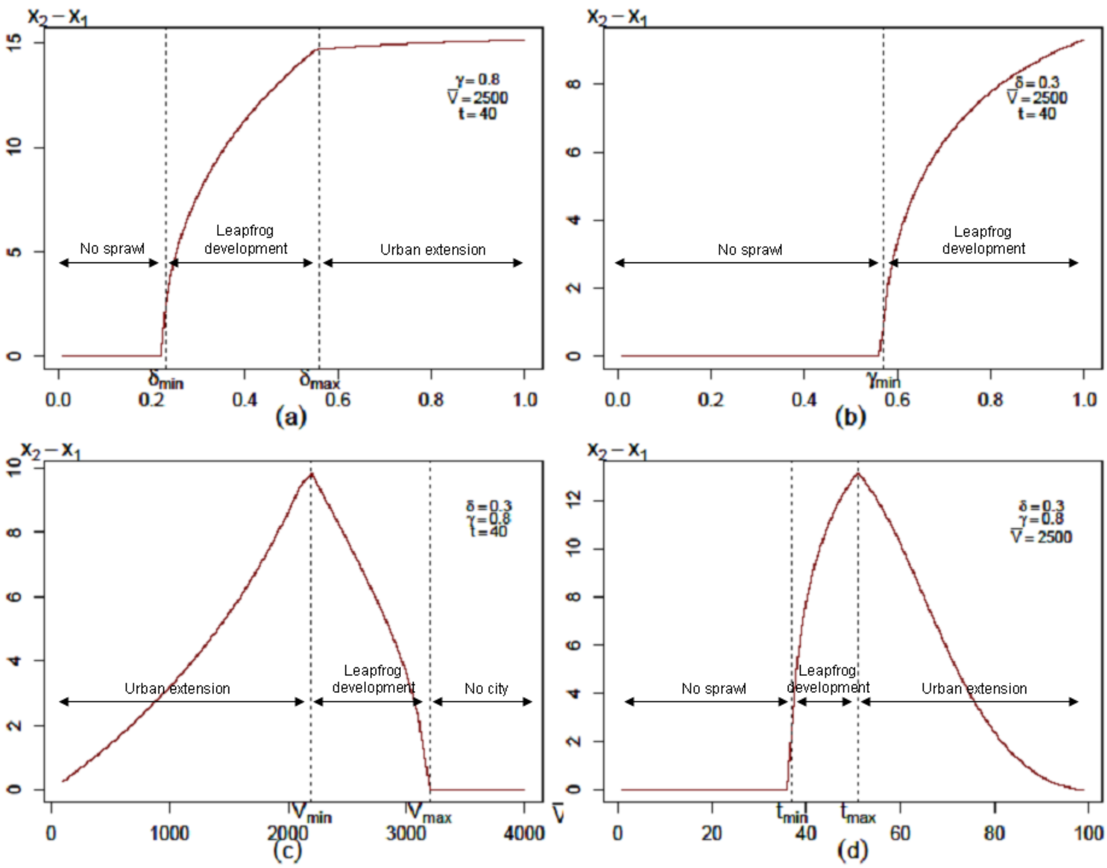

Figure 2: Impact of a change in δ, γ, V and t on spatial equilibrium patterns

grasslands) would be prone to sprawl under an urban extension scenario, whereas cities surrounded by agriculture characterised by a low capacity to produce amenities (for example, crop or livestock farming) may be subject to leapfrog development.

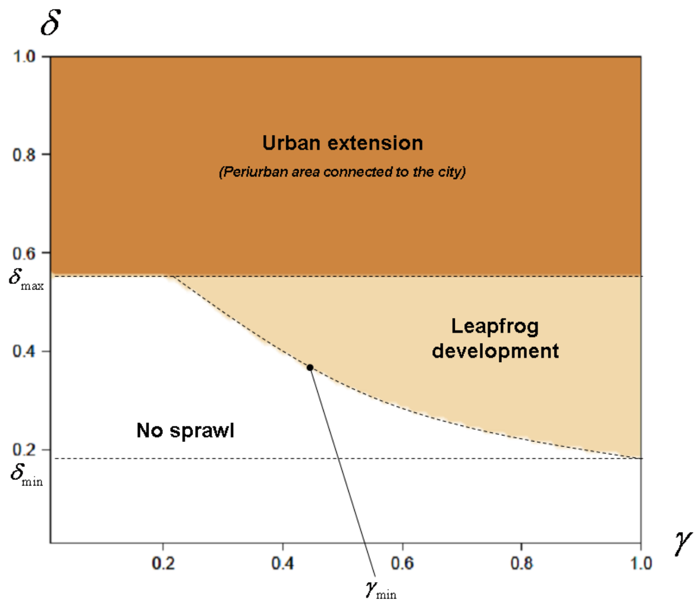

Figure 3: Urban development pattern depending on (δ, γ).

We now turn to the calibration of the equilibrium utility level. In our open-city case, migration in and out of the city is costless, so that the equilibrium utility level is exogenous, and the population size is endougenously determined by the model. From Eqs (9) and (12), we have $\partial r_u/\partial V < 0$ and $\partial r_p/\partial V < 0$. A change in the exogenous equilibrium utility level will have an impact on both urban and periurban households. Not only the size of the leapfrog area $(x_2 - x_1)$ will change, but also the location of the city boundary $\overline{x}$. A larger utility level decreases the urban bid-rent so that the city's boundary keeps getting closer to the CBD as V increases. Fig.2c presents the impact of a change in the equilibrium utility level on the size of the periurban area. While V is inferior to $V_{\min} = 2200$, the size of the periurban area keeps increasing. This is due to the fact that we have $x_1 = \overline{x}$ (case of urban extension) and the city boundary moves faster towards the CBD than x_2 does. From $V_{\max}$, the city "disappears", meaning that the urban bid-rent is too low for the city to continue to exist: that is all households prefer to migrate away from the city. All other parameters being equal, we have $V_{\max} \simeq 3200$. Between $V_{\min}$ and $V_{\max}$, we are in a leapfrog situation and the size of the periurban area decreases as V increases.

The final parameter that must be calibrated is transport costs for farmers. This parameter impacts on farmers' bid-rent functions, but also on periurban households' bid-rent via the level of k^* and thus the level of amenities provided. An increasing t lowers the farmer's bid-rent and increases the periurban household's bid-rent (from Eqs. (3) and (12), we have $\partial r_a/\partial t < 0$ and $\partial r_p/\partial t > 0$). Therefore, t impacts on the location of the urban fringe, $\overline{x}$ moving further away from the CBD when farmers' transport costs increase. t also impacts on the size of the periurban area. Fig.2d shows that up to a minimum threshold $t_{\min} = 37$, no sprawl occurs. This is due to the fact that farms become more intensive when transport costs to the city are lower $(\partial k/\partial t < 0)$. Therefore, the amenity level decreases and periurban households have no incentive to settle. Over $t_{\min}$, as the urban fringe keeps moving further away from the CBD, leapfrog development occurs. The increase in transport costs for

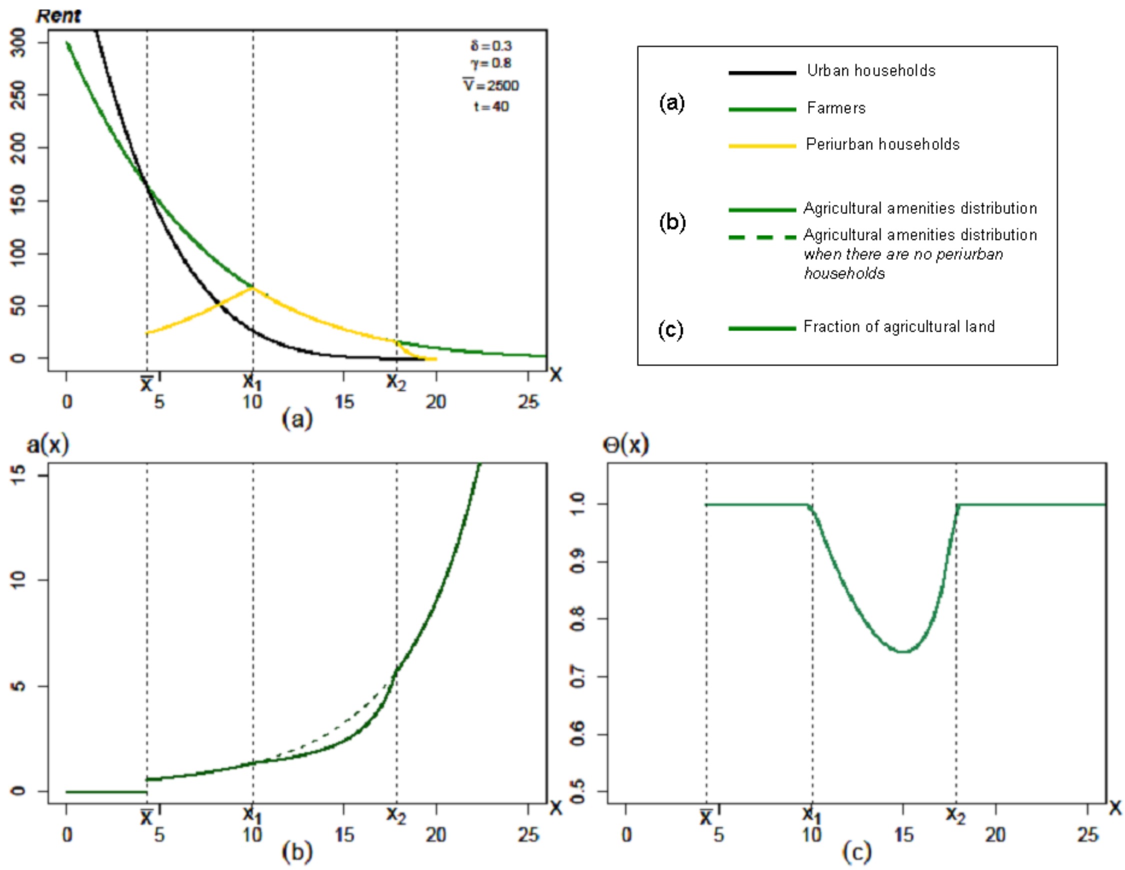

Figure 4: Spatial equilibrium pattern in the case of leapfrog development

farmers leads to a reduction in their use of production inputs, leading to a higher level of amenities. Finally, the maximum threshold $t_{\max} = 51$ indicates the moment when the periurban area joins up with the city, leading to urban extension. From this threshold, the size of the periurban area declines for two reasons: the first is the fact that it is limited by w/τ over which households would exceed their incomes. The second is the fact that urban fringe keeps moving further away from the CBD and getting closer to w/τ.

From here, we set $\gamma = 0.8$, $\delta = 0.3$, $V = 2\ 500$ and $t = 40$€. These four values ensure that we place ourselves in a potential leapfrog development context (see Fig.4a). Note that within the leapfrog area, the level of amenities is lower than it would be without any periurban households (see Fig.4b). Our model allows us to specify the fraction of land which is left to agricultural use in equilibrium (see Fig.4c). With our given parameters, the fraction of agricultural land within the leapfrog area decreases down to 74% below which too many amenities would be destroyed, so that no more periurban households will be incited to settle.

3.2 Parameter sensitivity

Now that all of the parameters are known, we propose to test the sensitivity of the remaining parameters. An increase in the level of household income is expected to make urban and periurban households increase their bid-rent, leading the city to expand but also producing a larger urban leapfrog area (from Eqs (9) and (12), we have $\partial r_u/\partial w > 0$ and $\partial r_p/\partial w > 0$). The agricultural bid-rent remains unchanged. Fig.5a shows how a positive variation in w moves $\overline{x}$ away from the CBD ($\overline{x}''$) and increases the size of the leapfrog area $(x_2 - x_1)''$; while a negative variation in income leads to a smaller city ($\overline{x}'$) and to a situation where no sprawl is observed. Note, that if the level of income decreases too far, all things being equal, the existence of the city is no longer assured, as condition (18) is not respected. The minimum value for w (all other parameters being fixed) can be calculated (see Appendix A for details). Here, we have $w_{\text{lim}} = 18257$€.

An increase in households' transport costs intuitively has the opposite effect, as it is equivalent to a negative variation in the level of their income. Increased transport costs for households will lead to a smaller city ($\overline{x}$ will decrease) and will reduce leapfrog development, the agricultural bid-rent remaining unchanged.

The final parameter influencing the urban bid-rent is the share of expenditures dedicated to housing β: from Eq (9), we have $\partial r_u / \partial \beta < 0$. An increase in β leads to lower urban and periurban bid-rents, making the city smaller and reducing the size of the leapfrog area. It has the same effect as a positive variation in transport costs.

The impact of the changing price of agriculture goods on the farmers' bid-rent is similar to the impact of changing incomes for households. As p increases, the farmers' bid-rent will increase and conversely, the periurban bid-rent will decrease. This is because a higher price for agricultural output will lead to further intensification of farms and thus a lower level of agricultural amenity. From Eqs (4) and (12), we have $\partial r_a / \partial p > 0$ and $\partial r_p / \partial p < 0$. Fig.5b shows how a small variation of p can influence the relative position of bid-rents. As both farmers and periurban households' bid-rents change, not only will the size of the leapfrog area change, but also the location of the city's boundary. An increase in the price of agricultural goods leads to a smaller city and a reduction in leapfrog development. Just as in the case of a change in income, we note the existence of a critical value for p over which condition (18) is no longer satisfied, meaning that the city cannot exist. Therefore, all things being equal, p must remain below $p_{\text{lim}} = 2112$€ in our case.

Conversely, compared to the price of agricultural outputs, the price of non-land inputs will have a negative impact on the farmer's bid-rent, but a positive one on the periurban bid-rent. If their price increases, a farmer will tend to substitute his non-land inputs with land inputs and therefore "*extensify*" his farm, providing a higher level of amenities. From Eqs (4) and (12), we have $\partial r_a / \partial p_k < 0$ and $\partial r_p / \partial p_k > 0$. The urban bid-rent remains unchanged. Thus, an increase of p_k will lead to a larger city and a larger leapfrog area.

Finally, the elasticity of the production factor k has the same effect as p_k, meaning that if α increases, the farmer's bid-rent decreases and the periurban bid-rent increases. In other words, an increase of α leads to a larger city and a larger leapfrog area, as shown in Fig.5c.

4 Land tax policies and urban sprawl

In this section, we discuss the impacts of the introduction of a property tax system. We assume that public authorities set two types of property tax, one for the land used for housing and one for agricultural land. We test the effect of these taxes on spatial equilibrium.

Introduction of land taxes in our model In our model, we denote the introduction of land taxes by θ_h, the property tax rate paid by periurban households. Thus, we obtain the following bid rent function:

$$r_p^*(x) = \left[\frac{\beta^\beta (1-\beta)^{1-\beta} (w - \tau x)}{V} \right]^{\frac{1}{\beta}} \frac{a(x)^{\frac{\gamma}{\beta}}}{(1+\theta_h)} \tag{27}$$

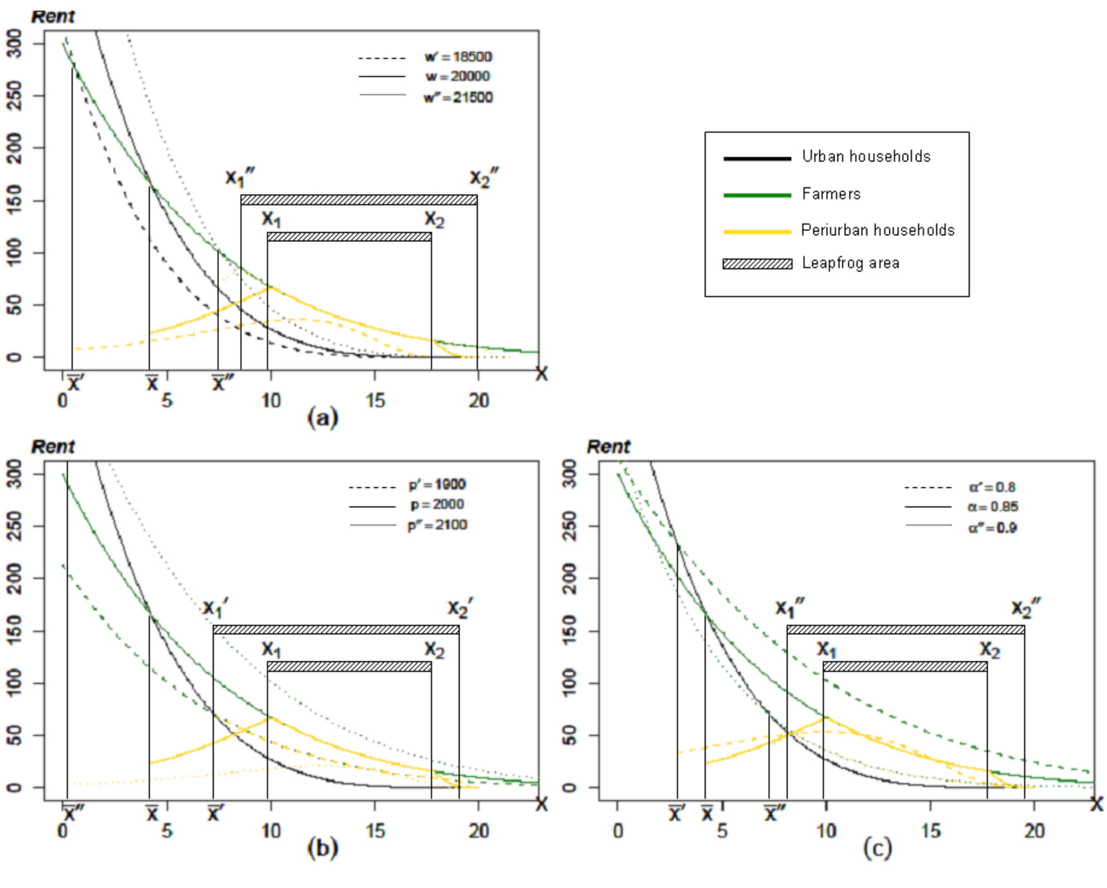

Figure 5: Urban development patterns following changes in (a) income, (b) agricultural good price and (c) elasticity of production factor k.

We note that the property tax rate has a negative effect on periurban households' bid-rent function. We also assume that the government applies a tax θ_a on agricultural land. The farmers' bid rent function becomes

$$r_a^*(x) = A(1-\alpha)\left(\frac{\alpha A}{p_k}\right)^{\frac{\alpha}{1-\alpha}} \frac{(p-tx)^{\frac{1}{1-\alpha}}}{(1+\theta_a)} \tag{28}$$

This tax has a negative effect on the farmer's bid-rent function. Therefore, the level of θ_a is also expected to have an impact on the location of the city's boundary $\overline{x}$. In the following we will study the effects of a land tax system on urban development.

Impact on spatial development patterns We test the sensitivity of spatial equilibrium with respect to variations in θ_h and θ_a. As expected, we see that, to curb urban sprawl, the government should tax more housing land than agricultural land. Fig.6 depicts the spatial development impacts of variations in θ_h and θ_a. We observe that θ_h and θ_a have opposite effects on spatial equilibrium patterns. While the first one leads to a smaller leapfrog area ($(x_2 - x_1)$ decreases), the second leads to a more expanded city and more developed leapfrog area.

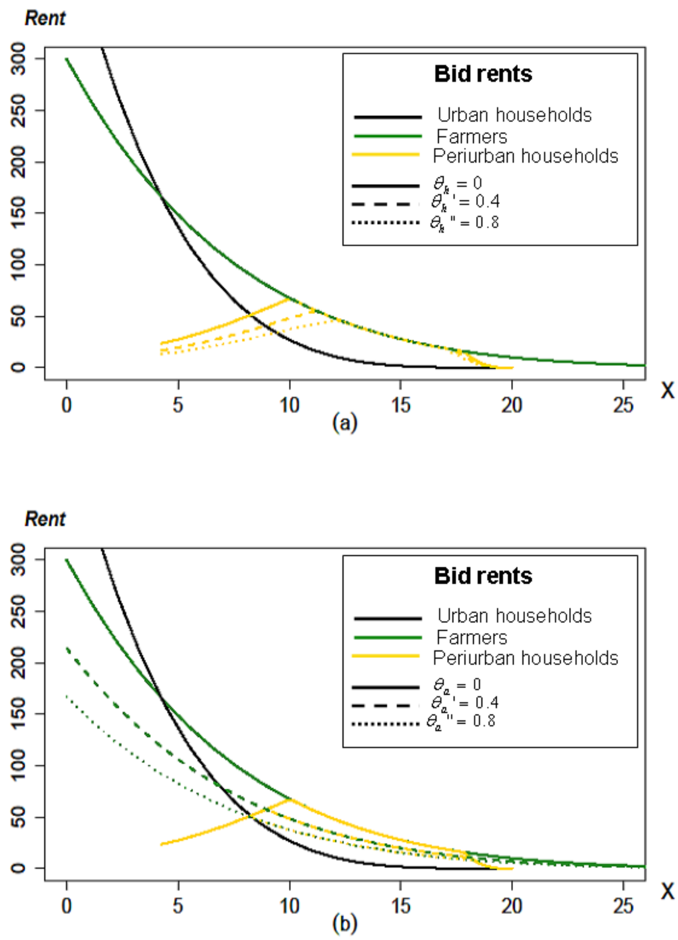

Figure 6: Impact of the introduction of land taxes on the relative position of bid-rent functions

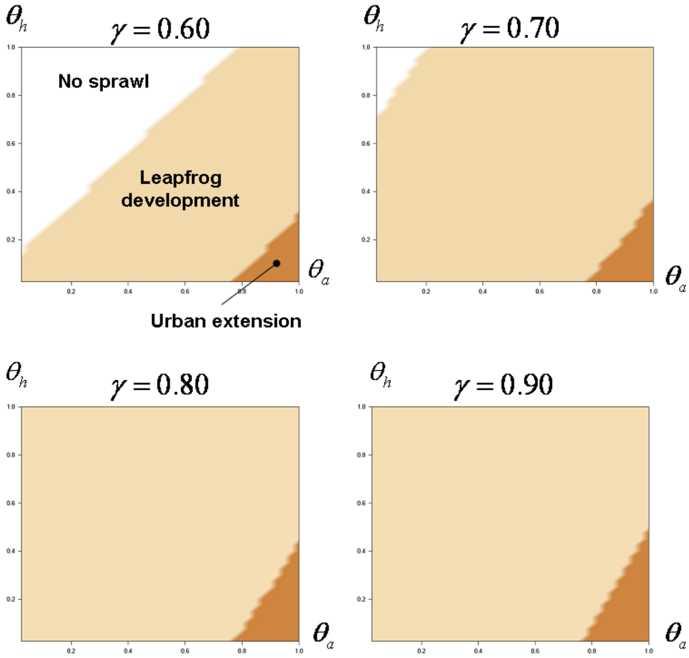

Figure 7: Impact of a change of θ_h and θ_a on the development pattern, at different levels of preferences.

Our model therefore shows how a land tax policy can be considered as a means of limiting leapfrog development. We observe in Fig.7 that from a given threshold, the property tax rate on periurban households, combined with low land taxation for agriculture, can prevent them from outbidding farmers. On the contrary, a high land tax for farmers combined with a low land tax for periurban households will encourage leapfrog development. However, this threshold depends to a great extent on the level of household preferences. For example, when $\gamma = 0.60$, the land tax rate for households must be greater than 0.15 (combined with a low tax rate for farmers) in order to limit leapfrog development. But when $\gamma = 0.70$, the same combination of land taxes is not effective at preventing leapfrog development, indeed θ_h must be greater than 0.7.

Similary, we observe in Fig.8 that the capacity of agriculture to provide amenities also has a strong influence on the optimal combination of land taxes. When the agricultural activity is characterised by a low capacity to provide amenities, for example $\delta = 0.20$, if the farmers' land tax rate is less than 0.4, there is no need to tax households in order to limit leapfrog development. However, as δ increases, then the households' tax rate must also increase, until we reach a situation where no land tax combination can curb leapfrog development (the case of $\delta = 0.30$ and $\delta = 0.35$ in Fig.8), all other things being equal.

From our results, we conclude that the effects of a land tax policy are highly dependent on household preferences for agricultural amenities and on the capacity of agriculture to provide them.

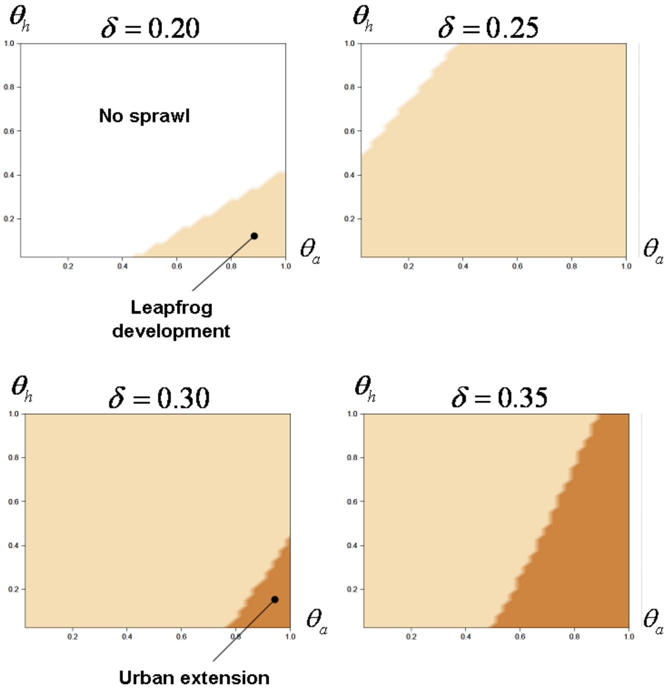

Figure 8: Impact of a change of θ_h and θ_a on the development pattern, at different levels of capacity for agriculture to provide amenities.

5 Conclusion

The purpose of this paper is to formally examine the relationship between urban spatial structure and agriculture. To highlight the importance of agricultural amenities, a monocentric city model has been developed which explicitly considers the behaviour of farmers *à la* von Thünen and identical households working in a predetermined CBD. Equilibrium is reached through a competitive land market. By endogenising agricultural amenities, we offer an intuitive explanation of the role of agriculture in the explanation of urban sprawl.

The results of the model illustrate potential variations in urban structure dependent on the nature of the farms and their distance from the city. Thus, farms close to the city tend to be relatively intensive, generating a low level of agricultural amenities. However, further away from the city, the rural landscape is characterised by a more extensive agriculture, which provides a relatively high level of amenity.

Some households enjoy living close to agricultural amenities and accept the associated long commute to work. When the households' bid-rent function is higher than that of farmers, leapfrog development is more likely to occur. What makes this scenario possible is the existence of a high level of amenities in the area of extensive agriculture, far from the city. In order to simulate the bid-rent curves, we calibrated our model using data relating the French context. For each of our parameters, we determined the thresholds, minimum and maximum, which allow the occurrence of leapfrog development. When households have a high preference for agricultural amenities and when agricultural activity is characterised by an intermediate capacity to provide amenities, the occurrence of isolated urban areas through leapfrog development is more likely.

Obviously this mechanism may operate in the absence of any public policy. But the introduction of a land tax system, may limit the leapfrog development. Thus, to curb urban sprawl, the government should tax housing land at a greater rate than agricultural land. However, the effect of a land tax on spatial urban structure depends on household preferences with respect to amenities and the ability of agriculture to provide them. In certain cases, low taxes on land are shown not to suppress the basic mechanisms that cause leapfrog development.

In the same way as with taxation, this approach can be used to test other public policies that aim to control urban sprawl. But, needless to say, any public policy that ignores the spatial dimension of agriculture may exhibit the same limitations. However, zoning policies may produce different results since they alter the distribution of agricultural activities and amenities. These questions are left for future research.

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A Existence of an urban boundary

We show that at the equilibrium, the households' and farmers' bid-rent curves intersect at least once within the interval $[0, \underline{x}]$, where $\underline{x} = \frac{p}{t}$ is the location from which agricultural activity stops.

Let's first analyze both bid-rent functions:

$$\begin{aligned}
\frac{\partial r_u^*}{\partial x} &= -\frac{1}{\beta}\tau \left[\frac{\beta^\beta (1-\beta)^{1-\beta}}{V}\right]^{\frac{1}{\beta}} (w-\tau x)^{\frac{1}{\beta}-1} < 0 \text{ at any } x < \frac{w}{\tau} \\
\frac{\partial^2 r_u^*}{\partial x^2} &= \frac{1}{\beta}\tau^2 \left[\frac{\beta^\beta (1-\beta)^{1-\beta}}{V}\right]^{\frac{1}{\beta}} (w-\tau x)^{\frac{1}{\beta}-2} > 0 \text{ at any } x < \frac{w}{\tau}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial r_a^*}{\partial x} &= -\frac{1}{(1-\alpha)} tA(1-\alpha) \left(\frac{\alpha A}{p_k}\right)^{\frac{\alpha}{1-\alpha}} (p-tx)^{\frac{1}{1-\alpha}-1} < 0 \text{ at any } x < \frac{p}{t} \\
\frac{\partial^2 r_a^*}{\partial x^2} &= \frac{1}{(1-\alpha)} t^2 A(1-\alpha) \left(\frac{\alpha A}{p_k}\right)^{\frac{\alpha}{1-\alpha}} (p-tx)^{\frac{1}{1-\alpha}-2} > 0 \text{ at any } x < \frac{p}{t}
\end{aligned}$$

Both bid functions are continuous, decreasing and convex within the intervals $\left[0, \frac{w}{\tau}\right]$ and $\left[0, \frac{p}{t}\right]$ respectively.

At equilibrium, we have:

$$r_u^*(0) = \left[\frac{\beta^\beta (1-\beta)^{1-\beta} w}{V}\right]^{\frac{1}{\beta}} \text{ and } r_a^*(0) = A(1-\alpha)\left(\frac{\alpha A}{p_k}\right)^{\frac{\alpha}{1-\alpha}} p^{\frac{1}{1-\alpha}}$$

$$\begin{aligned}
r_u^*(0) &> r_a^*(0) \Leftrightarrow \left[\frac{\beta^\beta (1-\beta)^{1-\beta}}{V}\right]^{\frac{1}{\beta}} w^{\frac{1}{\beta}} > A(1-\alpha)\left(\frac{\alpha A}{p_k}\right)^{\frac{\alpha}{1-\alpha}} p^{\frac{1}{1-\alpha}} \\
&\Leftrightarrow \boxed{w > \Omega p^{\frac{\beta}{(1-\alpha)}}}
\end{aligned}$$

where $\Omega = \frac{A^{\beta}(1-\alpha)^{\beta}\left(\frac{\alpha A}{p_k}\right)^{\frac{\alpha\beta}{1-\alpha}}}{\left[\frac{\beta^{\beta}(1-\beta)^{1-\beta}}{V}\right]}$

We also have:

$$\begin{aligned} r_u^*(x) &= 0 \Leftrightarrow \left[\frac{\beta^{\beta}(1-\beta)^{1-\beta}(w-\tau x)}{V}\right]^{\frac{1}{\beta}} = 0 \\ &\Leftrightarrow \boxed{x = \frac{w}{\tau}} \\ r_a^*(x) &= 0 \Leftrightarrow A(1-\alpha)\left(\frac{\alpha A}{p_k}\right)^{\frac{\alpha}{1-\alpha}}(p-tx)^{\frac{1}{1-\alpha}} = 0 \\ &\Leftrightarrow \boxed{x = \frac{p}{t}} \end{aligned}$$

The curves will intersect if and only if the following set of conditions is reached:

$$\begin{cases} r_u^*(0) > r_a^*(0) \\ x_{r_u^*(x)=0} < x_{r_a^*(x)=0} \end{cases}$$
$$\Leftrightarrow \begin{cases} w > \Omega p^{\frac{\beta}{(1-\alpha)}} \\ \frac{w}{\tau} < \frac{p}{t} \end{cases}$$

These conditions can be interpretated as:

1. The level of households' income must be relatively high enough, compared to the price of agricultural products.
2. The trade-off between urban and agricultural use can only be made within the interval $[0, \underline{x}]$, as from $\underline{x}$, all agricultural activity stops.

B Conditions for leapfrog development

Leapfrog appears when some households decide to settle within the agricultural area, meaning $x \in [\overline{x}, \underline{x}]$. Periurban households choose to live outside the urban area if the level of agricultural amenities is high enough. Their bid-rent function is given by:

$$r_p^*(x) = \left[\frac{\beta^{\beta}(1-\alpha)^{1-\beta}(w-\tau x)}{V}\right]^{\frac{1}{\beta}} a(x)^{\frac{\gamma}{\beta}}$$

We assume that leapfrog is a fragmented pattern of urban development, meaning that there is an intermediate area used for agriculture only. We consider the periurban area as an area where both households and farmers can locate.

$$P = \left\{x \in [\overline{x}, \underline{x}] \mid r_p^*(x) = r_a^*(x)\right\}$$

Leapfrog development is defined when the city and the periurban area are totally disconnected sets. In other words, there is a $x_1 < x_2 < \underline{x}$, so that for all $x \in [\overline{x}, x_1] \cup [x_2, \underline{x}]$, we have $r_a^*(x) > r_p^*(x)$, and for all $x \in [x_1, x_2]$, we have $r_p^*(x) = r_a^*(x)$.

According to our definition of leapfrog, at $\overline{x}$, we have:

$$r_p^*(\overline{x}) < r_a^*(\overline{x}) \Leftrightarrow r_u^*(\overline{x})a(\overline{x})^{\frac{\gamma}{\beta}} < r_a^*(\overline{x}) \Leftrightarrow \boxed{a(\overline{x}) < 1}$$

For the farmers to bid periurban households up at the city border, the amenity level must be inferior to 1.

$$\begin{aligned} a(\overline{x}) &< 1 \Leftrightarrow \delta \left[\alpha \frac{A\left(p - t\overline{x}\right)}{p_k}\right]^{-\frac{1}{1-\alpha}} < 1 \\ &\Leftrightarrow \delta < \delta_{\max} \end{aligned}$$

where $\delta_{\max} = \left[\alpha \frac{A(p-t\overline{x})}{p_k}\right]^{\frac{1}{1-\alpha}}$. As soon as δ gets larger than $\delta_{\max}$, leapfrog development as we define it (i.e. a fragmented development pattern) can't occur.

Immediately after the urban fringe, land is only used for agriculture if the amenity level is not attractive enough for periurban households. However, as we move further away from the center, the amenity level gets higher, which makes rural households' bid-rent function increase again.

C Characteristics of the leapfrog area

An intersection between farmers' bid-rent and periurban households' bid-rent occurs when $r_p^*(x) = r_a^*(x)$. Recalling that $r_p^*(x) = r_u^*(x)a^*(x)^{\frac{\gamma}{\beta}}$, we can derive the equilibrium level of agricultural amenities inside the periurban area:

$$\begin{aligned} r_a^*(x) &= r_p^*(x) \Leftrightarrow r_a^*(x) = r_u^*(x)a^*(x)^{\frac{\gamma}{\beta}} \\ &\Leftrightarrow a^*(x) = \left(\frac{r_a^*(x)}{r_u^*(x)}\right)^{\frac{\beta}{\gamma}} \\ &\Leftrightarrow a^*(x) = \left[\frac{A(1-\alpha)\left(\frac{\alpha A}{p_k}\right)^{\frac{\alpha}{(1-\alpha)}}(p-tx)^{\frac{1}{(1-\alpha)}}}{\left[\frac{\beta^\beta(1-\beta)^{1-\beta}}{\overline{V}}\right]^{\frac{1}{\beta}}(w-\tau x)^{\frac{1}{\beta}}}\right]^{\frac{\beta}{\gamma}} \end{aligned}$$

We can also define the periurban households density at equilibrium inside the periurban area, from Eqs. (7) and (22):

$$\begin{aligned} \rho_p^*(x) &= \frac{1}{q_h^*(x)}\left(1 - \frac{k^*(x)}{\delta}\left(\frac{r_a^*(x)}{r_u^*(x)}\right)^{\frac{\gamma}{\beta}}\right) \\ \rho_p^*(x) &= \frac{1}{\beta}\left[\frac{\beta^\beta\left(1-\beta\right)^{1-\beta}}{\overline{V}}\right]^{\frac{1}{\beta}}(w-\tau x)^{\frac{1-\beta}{\beta}}\left[1 - \frac{1}{\delta}\frac{\left[A\left(1-\alpha\right)\right]^{\frac{\beta}{\gamma}}\left(\alpha\frac{A}{p_k}\right)^{\frac{\alpha\beta+\gamma}{\gamma(1-\alpha)}}}{\left[\frac{\beta^\beta(1-\beta)^{1-\beta}}{\overline{V}}\right]^{\frac{1}{\gamma}}}\left(\frac{(p-tx)^{\frac{\beta+\gamma}{(1-\alpha)}}}{(w-\tau x)}\right)^{\frac{1}{\gamma}}\right] \end{aligned}$$

From which we can derive the fraction of agricultural land inside the periurban area:

$$\Theta^*(x) = 1 - \rho_p^*(x) q_h^*(x)$$

$$\Theta^*(x) = \frac{1}{\delta} \frac{\left[A\left(1-\alpha\right)\right]^{\frac{\beta}{\gamma}} \left(\alpha \frac{A}{p_k}\right)^{\frac{\alpha\beta+\gamma}{\gamma(1-\alpha)}}}{\left[\frac{\beta^{\beta}(1-\beta)^{1-\beta}}{\overline{V}}\right]^{\frac{1}{\gamma}}} \left(\frac{(p-tx)^{\frac{\beta+\gamma}{(1-\alpha)}}}{(w-\tau x)}\right)^{\frac{1}{\gamma}}$$